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ON *P'P'* AND RELATED WAVES.

By

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(With 5 figures.)

Zusammenfassung: Die kurzperiodischen Vertikalseismographen von BENIOFF, die in dem südkalifornischen Stationsnetz registrieren, haben mehrfach etwa $^1/_2$ Stunde, in einigen Fällen auch $^3/_4$ Stunde nach bestimmten Erdbeben, besonders solchen mit tiefen Herden, eine oder mehrere Phasen aufgezeichnet, deren Amplituden gelegentlich die der direkten Longitudinalwellen erreichen und mehrfach in den Monatsberichten von Pasadena als Nachbeben angeführt sind. Das Studium dieser Wellen ergab, daß es sich fast stets um drei bestimmte Phasen handelt (vgl. Tab. 1), von denen die erste als *P'P'*, die letzte als *P'P'P'* identifiziert wurde, d. h. als die einmal bzw. zweimal an der Erdoberfläche reflektierte Longitudinalwelle, die jedesmal auf ihrer Bahn durch das Erdinnere den Kern passiert hat. Die unter b in Tab. 1 angeführte Welle ist wahrscheinlich die Welle *ScPcPP'*[3]) (genauer geschrieben: *ScPcPPcPcP*), d. h. eine Welle ähnlich *P'P'*, die jedoch auf dem ersten Stück vom Herd bis zur Kerngrenze transversal gelaufen ist. Alle diese Wellen haben in der Gegend, in der sie beobachtet wurden, theoretisch einen Brennpunkt. Nähere Angaben hierüber befinden sich in Tab. 2, wo weitere derartige Wellen angeführt sind. Daß letztere nicht beobachtet wurden, liegt wohl zum Teil daran, daß in Pasadena nur sehr wenige geeignete Registrierungen, besonders solche von Beben mit großen Herdtiefen, aus den in Frage kommenden Herddistanzen vorhanden sind. Die Zeitdifferenzen *P'P'—P* und *P'P'P'—P* hängen sowohl den Beobachtungen wie der Theorie nach nicht wesentlich von der Herdtiefe ab, ändern sich dagegen stark mit der Herdentfernung, so daß sie zu deren Bestimmung auch im Falle von Herden mit außergewöhnlich großer Tiefe besonders geeignet sind.

I. Introduction.

The new vertical seismographs (1) designed by Mr. BENIOFF for routine use at this Laboratory can be set up to give records with high magnification for periods of about one second, with considerably less magnification for periods over 4 seconds. Instruments of this type

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[3]) Hier und in Zukunft werden die Striche über den Symbolen, die Brechung anzeigen sollen, weggelassen, da hierdurch keinerlei Verwechslungen möglich sind.

($T_0 = 0.5$, $T_1 = 0.2$ or 1.5) are in service at all seven stations of the southern California network.

Such seismograms give excellent detail in the preliminary phases of teleseisms, while the surface waves are either not recorded at all, or with such small trace amplitudes that they do not interfere with the record of the preliminaries. The seismograms of deep-focus earthquakes are especially satisfactory.

When a strong teleseism is followed by aftershocks, records of this type make it easy to pick out the beginning of each separate shock. Numerous cases of the kind are to be found in our files; but it has recently appeared that some supposed instances are spurious, being due to an entirely different phenomenon. This forms the principal subject of the present paper.

II. Observations on bodily waves with long travel times.

The occurrence of three strong deep-focus earthquakes at widely separated origins on August 29, Sept. 2, and Sept. 6, 1933, drew attention to the frequency of apparent aftershocks at intervals of about 30 and 45 minutes after the main shock. The suspicion at once arose that these were not separate earthquakes, but waves from the main disturbance which had traversed long paths. Accordingly, a search was made of all seismograms of Benioff instruments; with the result that 27 cases of the kind were identified, about evenly divided between normal and deep shocks. This covers the period since the first installation of Benioff instruments in February, 1931.

Table 1[1]) gives the following data for each shock: the date, time, and locality of origin; the depth h of focus in decimals of the radius r_0 of the earth (0.00 indicates a normal shock); the recording station (P, Pasadena; M, Mt. Wilson; R, Riverside; S, Santa Barbara; L, La Jolla; T, Tinemaha; H, Haiwee); the distance Δ in degrees; and the times of arrival of P and of the waves with long travel times mentioned above. The latter are separated into three groups representing three or more identifiable phases. A few readings of group a, marked with

[1]) For methods used in determining origin time, depth, and distance see the following paper. In addition to the readings of the Southern California stations, use was made of various station bulletins, as well as of the preliminary reports of the Jesuit Seismological Association and U. S. Coast and Geodetic Survey. We are especially indebted to N. H. Heck, Chief of the Division of Terrestrial Magnetism and Seismology of the latter organization, for a number of valuable readings of several recent shocks.

Table 1.

No.	Time of origin day	Time of origin h:m:s	Region	h/r_0	St.	Δ °	Times of arrival P m:s	a m:s	b m:s	c m:s
	1931									
1	March 2	02:18:22	SW Pacific	0.00	P	87	31:09	57:12	—	—
2	March 9	03:48:46	Japan	0.00	P	$74^1/_2$	00:28	27:53*	—	—
3	April 3	23:19:19	SW Pacific	0.10	P	79	30:28	57:13	—	—
4	May 20	02:22:50	E Atlantic	0.00	P	78	34:52	02:06	—	—
5	July 18	05:26:58	SW Bolivia	0.03	P	75	38:27	06:38	—	—
6	July 18	11:23:49	Kamchatka	0.01	P	$57^1/_2$	33:41	e 03:03	—	—
								i 03:31!	—	—
7	July 21	03:36:19	SW Pacific	0.03	P	87	48:57	14:54	—	—
8	Sept. 9	20:38:22	W Pacific	0.03?	P	86?	50:43	—	—	37:30
9	Nov. 2	10:02:57	Japan	0.00	P	87	15:44	41:45	—	—
	1932									
10	Jan. 9	10:21:42	SW Pacific	0.06	P	91	34:10	59:37	02:29?	20:25!
								01:15*	—	—
11	April 4	19:16:39	S of Japan	0.06	P	82	28:20	—	57:24	—
12	May 26	16:09:20	SW Pacific	0.11	P	85	21:08	i 47:10	49:59	08:00
								e 47:39	—	—
12	,,	,,	,,	,,	H	87	21:20	47:12	—	08:27
12	,,	,,	,,	,,	T	87	21:19	e 47:39	—	08:10
13	Sept. 23	14:22:06	Japan Sea	0.05	P	77	33:24	e 00:22		
								e! 00:42	03:35	20:20
								e 01:58*		
13	,,	,,	,,	,,	T	$75^1/_2$	33:14	e 00:43	03:48	—
14	Sept. 29	17:46:27	Kurile Is.	0.00	P	65	57:13	26:05	—	—
15	Nov. 2	11:03:18	SE Pacific	0.00	P	$58^1/_2$	13:11	42:55	—	—
15	,,	,,	,,	,,	T	61	13:32	42:51	—	—
16	Nov. 13	04:46:51	Japan Sea	0.05	P	78	58:19	24:55	28:11	45:16
16	,,	,,	,,	,,	M	78	58:17	—	—	45:22
16	,,	,,	,,	,,	H	77	58:11	25:12?	28:11	—
17	Nov. 26	04:23:57	N Japan	0.00?	P	74?	35:30	02:55	—	—
18	Nov. 29	11:11:02	Chile	0.00	P	78	23:03	50:26?	—	—
19	Dec. 9	08:34:52	Peru	0.01?	P	$64^1/_2$	45:28	14:21	—	—
19	,,	,,	,,	,,	T	66	45:39	14:12	—	—
	1933									
20	Febr. 23	08:09:16	N Chile	0.00	P	$70^1/_2$	20:34	48:29	—	—
20	,,	,,	,,	,,	T	74	20:46	48:36	—	—
20	,,	,,	,,	,,	R	70	20:29	48:28	51:04?	—
21	March 2	17:30:48	N Japan	0.00?	T	$73^1/_2$	42:22	e 10:04	13:26?	—
								i 10:15		

Table 1 (continued).

No.	Time of origin day	h:m:s	Region	h/r_0	St.	Δ °	P m:s	Times of arrival a m:s	b m:s	c m:s
21	March 2	17:30:48	N Japan	0.00?	M	75	e42:31	e 09.5?	13:45?	—
							i 42:36	i 09:59		
21	”	”	”	”	R	$75^1/_2$	e42:35	e 10:05	—	—
							i 42:40			
22	March17	15:55:25	Kamchatka	0.00	P	57	05:14	e 35:07	38:35?	—
								i 35:11	long	
								e 35:59*	waves	
22	”	”	”	”	T	55	04:59	35:18	—	—
22	”	”	”	”	R	$57^1/_2$	05:18	35:07!	—	—
23	July 9	12:30:36	Kurile Is.	0.00	P	68	41:37	e 09:58	—	—
								e 10:13		
23	”	”	”	”	M	68	41:39	10:15	—	—
23	”	”	”	”	R	$68^1/_2$	41:43	10:13	—	—
23	”	”	”	”	S	67	41:33	10:39	—	—
23	”	”	”	”	L	69	41:45	10:06	—	—
23	”	”	”	”	T	66	41:27	10:28	—	—
23	”	”	”	”	H	67	41:34	10:28	—	—
24	Aug. 29	14:52:30	NW Brazil	0.10	P	65	02:11	30:40	—	—
24	”	”	or	”	M	65	02:12	30:44	—	—
24	”	”	N Bolivia	”	R	$64^1/_2$	02:07	30:44	—	—
24	”	”	”	”	T	67	02:24	30:40	—	—
25	Sept. 2	16:41:08	S of Japan	0.07	P	83	52:57	19:22	22:01	—
25	”	”	”	”	L	$84^1/_2$	53:04	—	21:58	—
25	”	”	”	”	R	83	52:57	—	21:57	—
25	”	”	”	”	M	83	52:58	—	22:01	—
25	”	”	”	”	T	82	52:51	—	22:05	—
26	Sept. 6	22:08:25	SW Pacific	0.10	P	81	19:44	46:32	48:53	—
26	”	”	”	”	M	81	19:44	46:22	49.0	—
26	”	”	”	”	R	82	19:56	46:40	49:10	—
26	”	”	”	”	S	80	19:40	46:30	48:54	—
26	”	”	”	”	L	81	19:44	46:29	49:00	—
26	”	”	”	”	T	82	19:48	46:22	48:53	—
26	”	”	”	”	H	82	19:51	46:29	48:57	—
27	Oct. 2	15:29:18	Ecuador	0.00	P	51	38:22	09:31?	—	—
								10:05?	—	—

an asterisk, fall close to the calculated times for the already recognized phase *PPP* ($\Delta > 180°$); these will not be discussed further. The remaining readings define at least three travel-time curves, which correspond to no phases thus far mentioned in the literature.

The recorded amplitudes of these phases are surprisingly large, in some cases of the same order as that of P itself (cf. Fig. 1a). Their travel times show that they must have traversed very long paths; consequently there must be a considerable concentration of energy along these paths. In other words, for each of these phases there probably exists a focal point, such as is known for the phases P' and

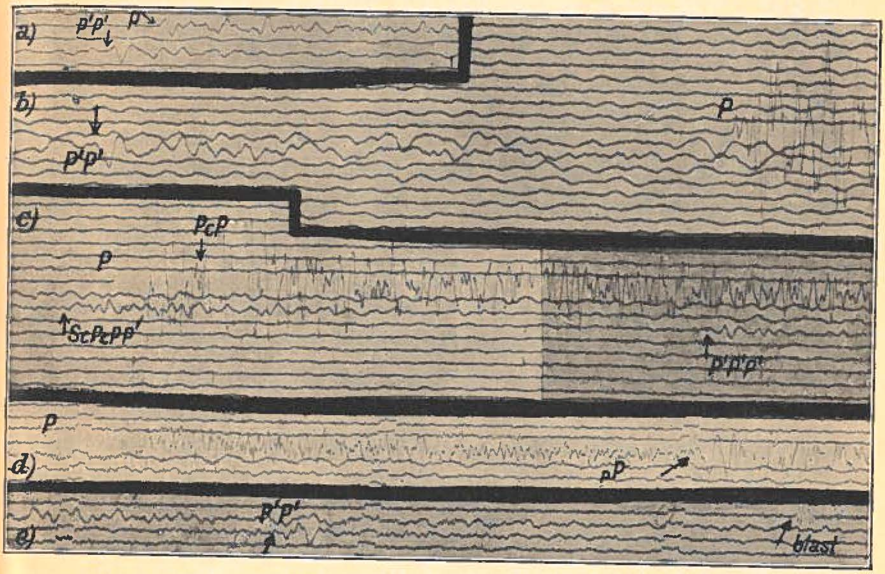

Fig. 1. Pasadena seismograms, Benioff vertical seismometer. $T_0 = 0.5$ sec.; $T_1 = 10$ sec. (a), 1.5 sec. (all others). Minute length 40 mm., reduced from 60 mm. Interval between successive lines about 15 min. (a) 1931 July 18 (No. 6, Table 1); (b) 1933 Febr. 23 (No. 20); (c) 1932 Nov. 13 (No. 16); (d, e) 1933 Sept. 6 (No. 26).

$ScPcP$[1]). The following table (Table 2) is calculated on the assumption of normal hypocentral depth. For each phase there is given the epicentral distance Δ of the focal point, and the travel-time to this point. The phases whose symbols occur on the same line in the two groups have the same travel-time curve for zero hypocentral depth; this is also true for those bracketed together on successive lines. For normal depth slight differences only can exist, which are negligible in calculation.

[1]) The bar indicating refraction is omitted from the phase symbol $ScPcP$, as no ambiguity can arise from this omission. The same applies to all similar symbols.

Table 2.

No.	Group I	Group II	Focal distance	Travel-time
1	$P'P'$	—	72°	$39^m\ 24^s$
2a	$P'PcPcS$	$P'ScPcP$	$83^1/_2$	42 41
2b	$ScPcPP'$	$PcPcSP'$		
3a	—	$P'ScPcS$	98	45 42
3b	—	$ScPcSP'$		
4a	$PcPcSScPcP$	$PcPcSPcPcS$	96	45.6
4b	$ScPcPPcPcS$	$ScPcPScPcP$		
5a	$PcPcSScPcS$	$ScPcPScPcS$	114	48 33
5b	$ScPcSScPcP$	$ScPcSPcPcS$		

All these waves have been twice refracted through the core, and once reflected at the surface of the earth. Only one other phase of this type is possible, namely *ScPcSScPcS*. This has been omitted from the tabulation, as it has no focal point.

Group II consists of cases in which a change from longitudinal to transverse waves, or *vice versa*, occurs at the surface reflection. As the angle of incidence is small in the cases considered, only a small percentage of the energy goes into the reflected ray. Accordingly, waves of Group II should show very much less energy than the corresponding waves of Group I, for which there is no such change in character on reflection.

Apart from diffracted waves, the travel-time curves extend only to smaller distances than the focal distance given. The travel-times all increase with decreasing distance. These times are easily calculated by the method already in use for calculating the times for *PP*, *PS*, etc. The data for the travel-times of *P′P′* are derived from those given for *P′* in "Handbuch der Geophysik", vol. IV, p. 217, by Gutenberg. All others are derived from Table 14 of "Über Erdbebenwellen, VII A" (2).

The observations under (a), Table 1, agree very well in general with the calculations for *P′P′*. The readings under (b) are identified with reasonable probability as *ScPcPP′*; but as only doubtful readings of this group are available for normal shocks, a correction for the hypocentral depth is necessary. No observations fit any of the other phases of Table 2; this may be due to the fact that comparatively few seismograms of strong recent shocks at the favorable distances are available at Pasadena, none of these being cases of deep focus.

The readings under (c) are identified as *P′P′P′*. For normal hypocentral depth this should have a focal point at about 72° epicentral distance, with a travel-time of about $59^m.1$. In this case the travel-time curve for undiffracted waves extends to increasing epicentral distance, with increasing times. Again, the observations all refer to cases of abnormal depth, so that a correction is required.

III. $P'P'$.

Figures 2 and 3 represent the observations of this phase. In Fig. 2 the travel-times are plotted directly against distance; in Fig. 3 there are

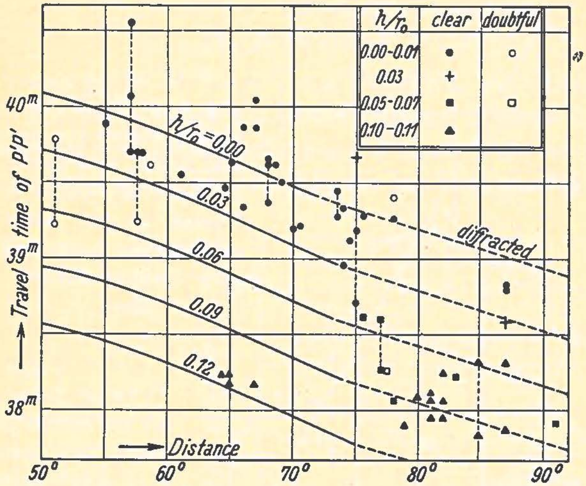

Fig. 2. Travel times of $P'P'$. Calculated curves for various depths, and observations of Table 1. Phases of the same seismogram connected by vertical dashed lines.

plotted the intervals $P'P'—P$. Data from various hypocentral depths are distinguished by the use of different symbols in plotting the points.

The scatter of the points in Fig. 3 is markedly less than in Fig. 2. Inspection shows that this is due to the fact that the interval $P'P'—P$ is nearly independent of the hypocentral depth, while the travel-times of both phases are of course shorter for deeper shocks.

The following Table (Table 3) gives, for three epicentral distances and three hypocentral depths, the amount by which the travel-time of P falls short of that for normal depth at the same distance; also the corresponding deficit for $P'P'$, which does not vary greatly in the range of distance considered. These quantities are calculated from the travel-times published by SCRASE (3).

Table 3.

Depth	P (normal) — P (deep) $\Delta = 50°$	$\Delta = 70°$	$\Delta = 90°$	$P'P'$(normal) — $P'P'$(deep)
0.03	17^s	23^s	17^s	23^s
0.06	35	41	38	46
0.09	52	60	54	68

It is clear that the differences for $P'P'$ are very nearly the same as those for P at the same hypocentral depth; so that the interval

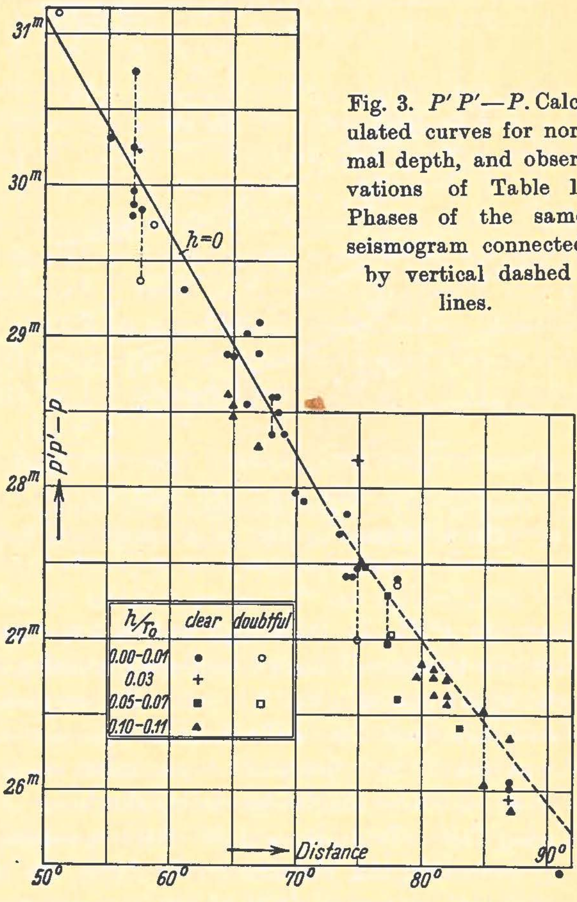

Fig. 3. $P'P'$—P. Calculated curves for normal depth, and observations of Table 1. Phases of the same seismogram connected by vertical dashed lines.

$P'P'$—P should in fact be nearly independent of this depth. The theoretical curve for normal depth is drawn in Fig. 3.

On Fig. 2 are drawn the travel-time curves for $P'P'$, as calculated from the data of "Handbuch der Geophysik", IV, 217, with the corrections for hypocentral depth just given (Table 3). The data fit the calculations very well, especially considering the uncertainties in connection with the deep-focus shocks. For normal shocks it is clear that the best observations are a few seconds earlier than the theoretical expectation. This is consistent with the fact that P' usually is a little earlier than the arrival times now in general use; also, it is possible that the reflection of $P'P'$ takes place at some discontinuity in the crust of the earth.

IV. *ScPcPP'*?

The readings (b) of Table 1 are identified, with some doubt, as *ScPcPP'*. The travel-times are plotted against distance in Fig. 4. As the intervals between these readings and P show nearly as much scatter as the times themselves, no second figure has been plotted. Except for

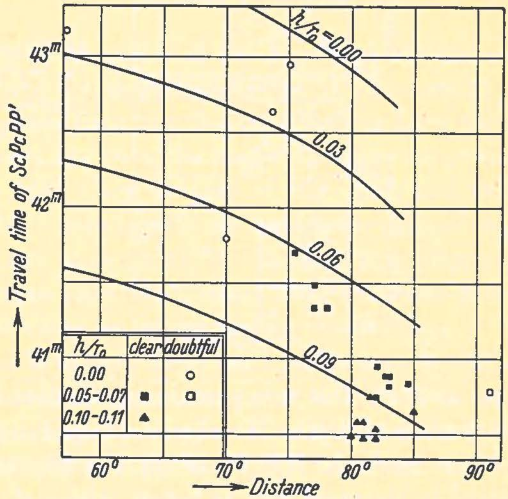

Fig. 4.
Travel times of *ScPcPP'*. Calculated curves for various depths, and observations of Table 1.

four doubtful cases in normal shocks, all the plotted points refer to very deep focus. This introduces additional uncertainty into the comparison with theory. The curves drawn in the figure are calculated from the same data as used in III. The agreement is, in general, good.

As this wave is transverse in the first segment of its path, the effect of increased hypocentral depth is greater than for P, P', or $P'P'$. The values corresponding to those at the right of Table 3 are 43, 86, and 127 seconds instead of 23, 46, and 68 respectively. Consequently

ScPcPP'—P is expected to depend on hypocentral depth, much more than *P'P'—P*.

Theoretically, the wave *ScPcPP'* from a deep source should be followed, at an interval of from a few seconds to one minute, by *P'PcPcS*. Though there is occasionally continued disturbance following the readings (b), there is no definite observation of a second arrival. Similar remarks apply to other theoretically possible phases, such as *pP'P'*, *sP'P'*, *pScPcPP'*, etc.

V. *P'P'P'*.

Although the data are exclusively from deep shocks, the identification of the readings under (c) as *P'P'P'* is nearly as good as that

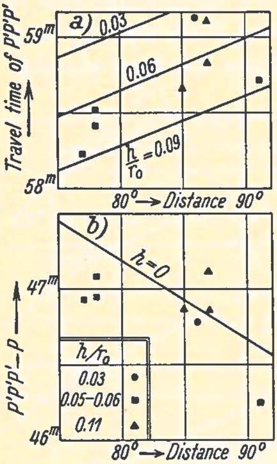

Fig. 5.
(a) Travel times of *P'P'P'*.
(b) *P'P'P'—P*. Calculated curves, and observations of Table 1.

of (a) as *P'P'*. This is in part due to the fact that *P'P'P'—P* is nearly independent of depth. The observed travel-times are plotted in Fig. 5a, and the intervals *P'P'P'—P* in Fig. 5b, together with curves calculated in the same way as before. Although the observations are few in number, the verification of theory is surprisingly good; the divergence from the theoretical times is only a small fraction of the whole travel-time, which is nearly one hour. In this case *P'P'P'—P* decreases with increasing distance, while *P'P'P'—O* increases.

VI. General remarks.

The newly identified phases promise to be of considerable value in determining both depth and epicentral distance of a recorded earthquake. As the intervals $P'P'—P$ and $P'P'P'—P$ are nearly independent of depth, these phases, when recorded, afford a check on the distance without first requiring that the depth be known. If the origin time is determined, the travel times of these phases give a measure of the depth. All this will be found in detail in the following paper.

A further valuable verification of distance and depth is afforded by the observation of these phases, with measurable amplitudes, only in the vicinity of their focal points. Thus $P'P'$ has in no instance been observed at less than 50° from the epicenter, although numerous records of strong normal earthquakes in this distance range are available at Pasadena. In normal shocks this phase records with large amplitudes only at distances from 55° to 75°; the reflection then takes place at 152° to 142°. In deep shocks the large amplitudes occur at somewhat larger distances, which agrees with theory. No $P'P'$ has been observed beyond 91°. There are similar limits for $ScPcPP'$ and $P'P'P'$; for the latter the reflections occur at intervals of about 146° to 150° in the few observed cases.

There are no observations of waves of the type here studied at distances less than 55° (except one doubtful $P'P'$ at 51°) or over 91°. However, no deep-focus records from the shorter distances have been written at Pasadena; and very few from the longer distances are available. Two seismograms at 135° show nothing of the kind; and this also appears to be true of a seismogram at 145°.8 reproduced by Scrase (4). Nevertheless, such phases may be recorded. For instance, $P'P'P'P'$ should have a focus near 145°; other focal distances have been given in Table 2, and there are many further possibilities.

Some readings of L and M in deep-focus earthquakes may actually be readings of $P'P'$, etc. Instances apparently occur in the publication by Scrase (4). When very sharp, these phases may be reported as separate shocks, as has sometimes occured in the Pasadena station bulletin.

References.

(1) Hugo Benioff, A new Vertical Seismograph. Bull. Seism. Soc. Amer. **22** (1932) 155. — (2) B. Gutenberg, Über Erdbebenwellen VIIa. Nachr. Ges. Wiss. Göttingen 1914. — (3) F. J. Scrase, The reflected waves from deep focus earthquakes. Proc. Roy. Soc. London (A) 132 (1931) 213. — (4) The characteristics of a deep focus earthquake. Philos. Trans. Roy. Soc. London (A) **231** (1933) 207.